Demystifying the 2:1 Ratio and the Stick-Slip Phenomenon

A Technical Whitepaper Explaining the Theory Behind the Binding Ratio and How it Relates to Stick-Slip

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One of the most frequently misunderstood principles regarding the use of plane bearings is something known simply as the 2:1 Ratio. Most engineers are taught there is this magic ratio (2:1) of allowable moment arm length to bearing length; which cannot be violated or the application will fail. This paper will explore the history behind the 2:1 Ratio, the mathematical theory supporting the rule, practical limitations of implementing the 2:1 Ratio and finally some simple troubleshooting steps which can be taken to overcome any problems.

INTRODUCTION

One of the most frequently misunderstood principles regarding the use of plane bearings is something known simply as the 2:1 Ratio. Most engineers are taught there is this magic ratio (2:1) of allowable moment arm length to bearing length; which cannot be violated or the application will fail. This paper will explore the history behind the 2:1 Ratio, the mathematical theory supporting the rule, practical limitations of implementing the 2:1 Ratio and finally some simple troubleshooting steps which can be taken to overcome any problems.

HISTORY

Since the 2:1 Ratio was first introduced to the marketplace by Pacific Bearing Company (PBC) in the 1990's, it has been quickly adapted by most self-lubricating, linear plane bearing manufacturers as one of the guiding principles regarding their use. In the 1980's, PBC invented and began to commercially produce the first plane bearing which were size interchangeable with linear ball bearings. This led to a new learning curve for engineers; which were primarily used to working with ball based systems or bronze bearings with higher coefficients of friction. The plane bearing systems excelled in many environments where recirculating ball based systems failed miserably; including: very hot or cold, dirty, high vibration, high static loads, non-lubricated and short stroke (<2 x bearing length). The plane bearings were not found to be an acceptable replacement for applications which required a low coefficient of friction, very high speeds or high moment loads. In fact, some applications with high moment loads were known to simply bind up and all motion would cease, or it would become jerky (also known as stick-slip motion). Early on, the assumption was that plane bearings could not handle the same moment load as an equivalent sized recirculating ball bearing. What engineers failed to realize is there was a geometric relationship which describes the allowable working space of plane bearings—that is until PBC published the 2:1 Ratio. Two decades later, some engineers *still fail* to realize the broad scope of this rule and that it actually applies to <u>all</u> linear motion systems, not just plane bearings! In addition, the 2:1 Ratio isn't always "2:1". Depending upon the unique criteria of each application, the actual ratio may be larger or smaller than 2:1. For the purposes of this whitepaper, the 2:1 Ratio will be referred to as the "Binding Ratio".

Definition

The "Binding Ratio" is officially defined as the maximum ratio of moment arm distance to bearing length which will not bind (prevent motion). The Binding Ratio is often displayed numerically as "X:Y," where "X" is the moment arm distance and "Y" is bearing length. Typically, the value of "X" is divided by "Y" so that the ratio can be expressed as "$^{X}/_{Y}$:1". In the specific case of this whitepaper, the ratio used is "2:1". The binding ratio can be theoretically defined using mathematics; however, several factors complicate practical implementation.

THEORY

One of the key principles behind the "Binding Ratio" is Sir Isaac Newton's 3rd Law of Motion: for every action there is an equal and opposite reaction. The remainder breaks down to basic statics and dynamics equations. When a force is applied to a bearing at some distance (D_1) away from the center of the bearing, a moment force is created. In order to resist the moment, two resulting forces are created at each end of the bearing. When these resulting forces are multiplied by the Coefficient of Friction, a drag force is created. At some point the drag force will surpass the drive force and motion will cease. This paper will show that the size of the drive force is irrelevant as the maximum moment arm (as a ratio to bearing length) is limited solely by the Coefficient of Friction. The remainder of this section will show the derivations and proof of this concept for two different situations.

The example will be where a single force (F_1) is being applied to a linear bearing system at a known distance (D_1). Assuming the bearing system and moment arm are a rigid body, motion will be in the same direction that F_1 is applied. It can also be assumed that linear bearing systems are designed to allow only one degree of freedom; which is the axis of motion. Figure 1, below, is PBC's standard mini-Rail product. Figure 2, below, shows the bearing with a force (F_1) being applied to it at a known distance (D_1) as well as the length of the bearing (L_1). For the purposes of keeping these equations simple, an assumption is made that the applied force (F_1) is being applied in a single plane and is parallel to the direction of travel. Figure 3, below, repeats the values shown in Figure 2, and then adds the equal and opposite reaction forces (F_2 & F_3); which are a result of the applied moment. Forces F_2 and F_3 are applied at points A and B, respectively.

Figure 1: Standard MR Product

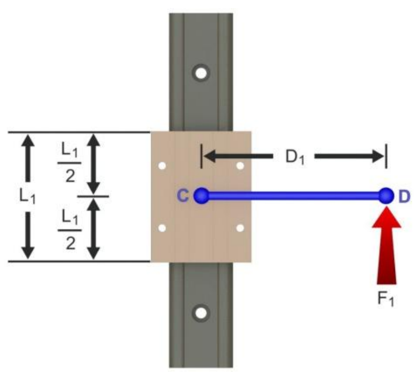

Figure 2: Force (F_1) is Applied

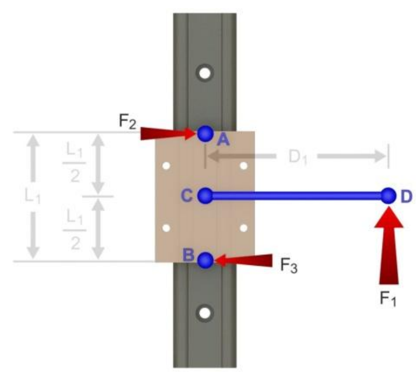

Figure 3: First Reaction Forces (F_2 & F_3)

Figure 4, top of next page, then shows the drag forces (F_4 & F_5); which are caused by the resulting forces (F_2 & F_3) multiplied by the coefficient of friction of the carriage versus the rail (μ) when the carriage is in motion. Figure 5, top of next page, shows the complete system and all forces acting upon the carriage.

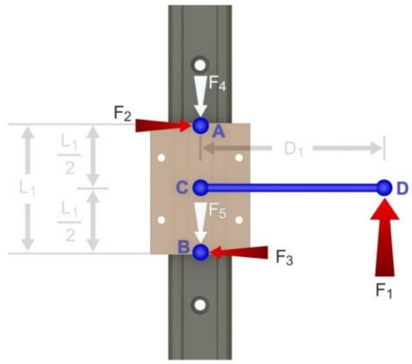

Figure 4: Second Reaction Forces (F_4 & F_5)

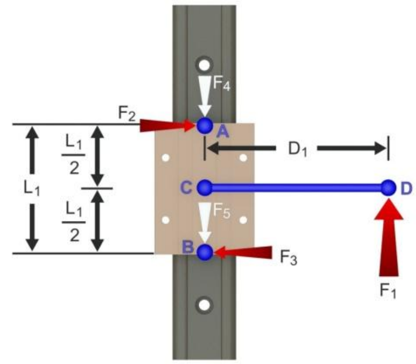

Figure 5: All Forces

Building upon Sir Isaac Newton's 1st Law of Motion (an object at rest will stay at rest until the vectored sum of all forces acting upon it is greater than zero), in order for the carriage to accelerate the applied force (F_1) must be greater than the sum of the drag forces (F_4 & F_5). In practice, there is an additional drag force due to the weight of the carriage plus the weight of the payload. For now, this additional resistance will be disregarded.

Equation 1: $$Acceleration\ (motion)\ > 0\ \rightarrow\ F_1 > F_4 +\ F_5$$

Equation 2: $$F_4 = \mu F_2\ \ and\ \ F_5 = \mu F_3$$

Equation 3: $$Acceleration\ > 0\ \rightarrow\ F_1 > \mu F_2 +\ \mu F_3$$

Equation 4: $$Acceleration\ > 0\ \rightarrow\ F_1 > \mu(F_2 +\ F_3)$$

To begin the statics equations, point C is chosen to be the fulcrum and all forces and moments are summed around this point. Figure 6, below, shows the lever arms resulting from the applied forces (F_1) and the resulting force (F_2 & F_3) with an origin of point C.

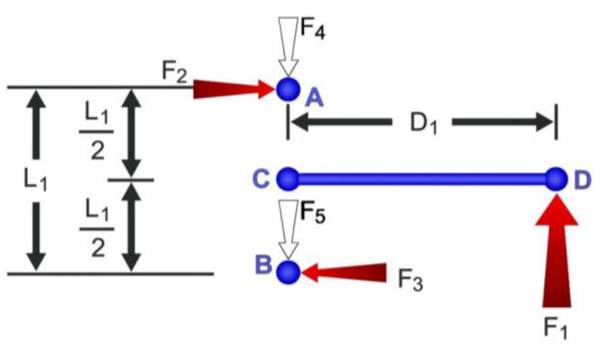

Figure 6: Free Body Diagram

Since linear bearing systems are designed to be rotationally stable, it is known that the sum of the moments (about point C) is equal to zero (Equation 5, below). Equations 6 – 10, below, show the simplification of this equation in order to solve for the resulting forces F_2 and F_3.

Equation 5: $$Rotationally\ Stable\ \rightarrow \sum M_Z = 0$$

Equation 6: $$\sum M_Z = 0 = (F_1 \times D_1) - \left(F_2 \times \frac{L_1}{2}\right) - \left(F_3 \times \frac{L_1}{2}\right)$$

Equation 7: $$(F_1 \times D_1) = \left(F_2 \times \frac{L_1}{2}\right) + \left(F_3 \times \frac{L_1}{2}\right)$$

Equation 8: $$(F_1 \times D_1) = \frac{L_1}{2}(F_2 + F_3)$$

Equation 9: $$\frac{2 \times F_1 \times D_1}{L_1} = (F_2 + F_3)$$

At the transition point, where the bearing transitions from binding to motion (or vice-versa), the acceleration value from Equation 4 transitions from a negative to a positive value. In order to solve for this, set the value to zero to find the specific transition point. The following group of equations is valid only at the transition point, where acceleration is equal to zero.

Equation 10: $$Acceleration = 0 \rightarrow F_1 = \mu(F_2 + F_3)$$ *Valid at Transition Point*

Equation 11: $$F_1 = \mu\left(\frac{2 \times F_1 \times D_1}{L_1}\right) = \frac{2 \times \mu \times F_1 \times D_1}{L_1}$$ *Valid at Transition Point*

Equation 12: $$\frac{F_1}{F_1} = 1 = \frac{2 \times \mu \times D_1}{L_1}$$ *Valid at Transition Point*

As mentioned before, in order for an application to be capable of motion, the driving force must be greater than the sum of the drag forces. Equation 12, above, shows that there is a relationship between the Coefficient of Friction (μ), bearing distance and moment arm distance. Rearranging Equation 12 will derive the three conditions that must be met in order for an application to be capable of motion. Equations 13-15, below, will separate the maximum allowable Coefficient of Friction (μ), the maximum allowable moment arm distance (D_1) and the minimum allowable bearing length (L_1).

Equation 13: $$L_{1\_min} = 2 \times \mu \times D_1 \rightarrow \boldsymbol{L_1} > 2 \times \mu \times \boldsymbol{D_1}$$ *Condition 1*

Equation 14: $$D_{1\_max} = \frac{L_1}{2 \times \mu} \rightarrow \boldsymbol{D_1} < \frac{\boldsymbol{L_1}}{\boldsymbol{2 \times \mu}}$$ *Condition 2*

Equation 15: $$\mu_{max} = \frac{L_1}{2 \times D_1} \rightarrow \boldsymbol{\mu} < \frac{\boldsymbol{L_1}}{\boldsymbol{2 \times D_1}}$$ *Condition 3*

At the beginning of this paper, the claim was made that the Binding Ratio is a pure derivative of the coefficient of friction and the relationship of the bearing length to moment arm distance. In order to better illustrate this point, a graph can be created showing what the maximum allowable Binding Ratio would be for any coefficient of friction. Figure 7, below, illustrates that the smaller the coefficient of friction, the larger the Binding Ratio can be. For this figure, Equation 14 was used and the bearing length, L_1, was set to a value of $L_1 = 1$. The X-axis shows different values for the coefficient of friction, μ, and the Y-axis shows the Binding Ratio. For illustration purposes, the lowest common coefficient of friction commonly found in linear motion systems today is in recirculating ball type linear guides where the coefficient of friction can be as low as μ = 0.001. Using Equation 14, and setting the bearing length to 1, the result is a Binding Ratio of 500:1! The highest coefficient of friction found in modern day plane bearings is μ = 0.5; however, most modern day materials have a value of μ = 0.1 - 0.25. Figure 7 graphically illustrates the importance of a low coefficient of friction for applications with a high moment load and a large moment arm distance.

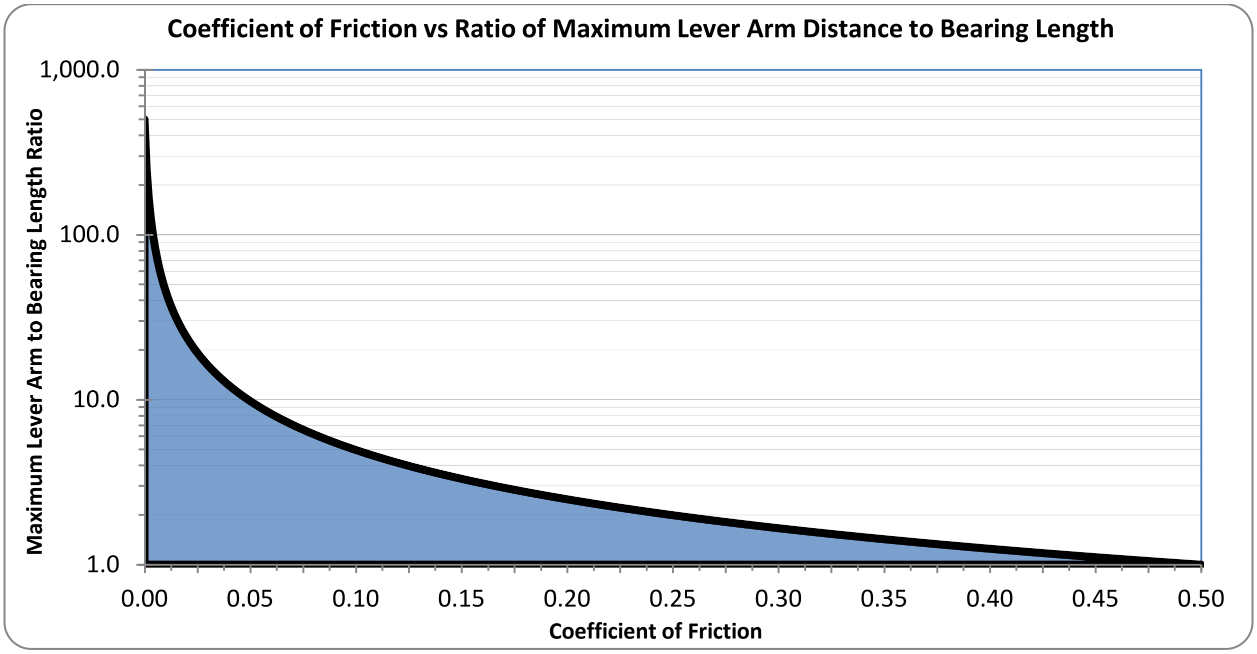

Figure 7: Coefficient of Friction vs. Ratio of Max Lever Arm Distance to Bearing Length

Assumptions

The assumption was made that all forces are applied in an axis which is parallel to the direction of travel. This eliminates the complication and redundancy of vector mathematics. In most practical applications, the force would actually be applied at some non-parallel angle which would require the force to broken down into its X-Axis, Y-Axis and Z-Axis vectors. The end result would be the same as shown herein. The only difference is that the formulas shown above would have to be repeated three times, once for each axis.

STICK-SLIP

One of the most frustrating and potentially devastating problems an engineer can face is stick-slip motion. It is particularly devastating because it is not planned for and can bring production to a standstill until the problem is solved. This paper will explore one of the causes for stick-slip. The previous section of this paper discussed the coefficient of friction (μ) as it was a single value. In reality, every material has *two* coefficients of friction: static (μ_s) and dynamic (μ_k). In general, the coefficient of static friction is greater than or equal to the coefficient of dynamic friction. Using Machinery's Handbook as a reference, a basic comparison of materials showed that, on average, the coefficient of dynamic friction was equal to 65% of the coefficient of static friction. This value will certainly be different for every material, so be sure to check the specific value for designed in materials.

Building upon what was learned in the previous section (the greater the coefficient of friction, the lower the Binding Ratio will be), the difference between the static and dynamic coefficient of friction means that there could possibly be some designs which will work when the system is already moving, but will not work when the system is at rest. Figure 7, below, plots the Binding Ratio for different static and dynamic coefficients of friction. Figure 8, below, is a detail view of Figure 7. There are two curves plotted in each figure. The top curve represents the Binding Ratio curve based upon the *static* coefficient of friction. The bottom curve represents the Binding Ratio for the *dynamic* coefficient of friction based upon the assumption that the dynamic coefficient of friction is 35% less than the static coefficient of friction. The blue area at the bottom of the plot is the "free motion zone" where motion will not theoretically be interrupted by binding due to the Binding Ratio. The red area is the area between the curves of the Binding Ratio for the static and dynamic coefficients of friction. The white area above the curves is the "no motion" zone where it is likely that motion will not occur. It is the claim of the author that stick-slip is more likely to occur for applications where the Binding Ratio is in the red zone (the area between the curves).

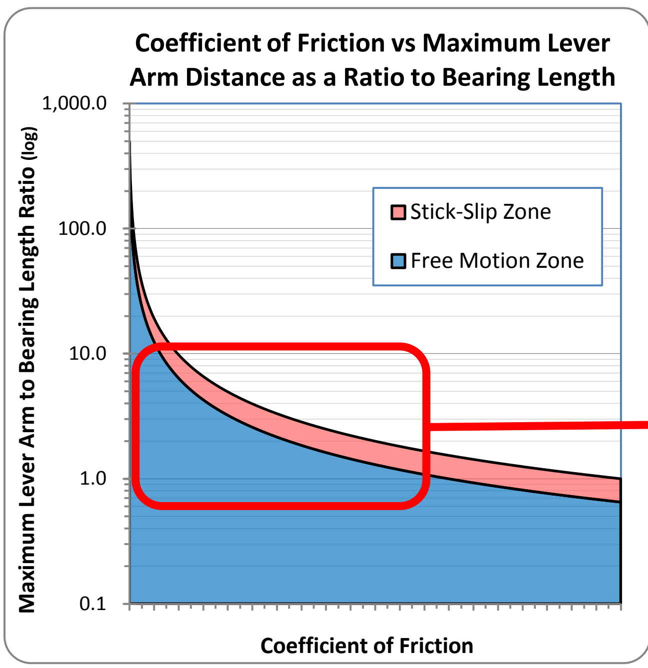

Figure 7: Coefficient of Friction vs. Max Lever Arm Distance for Bearing for Present Day Bearing Systems

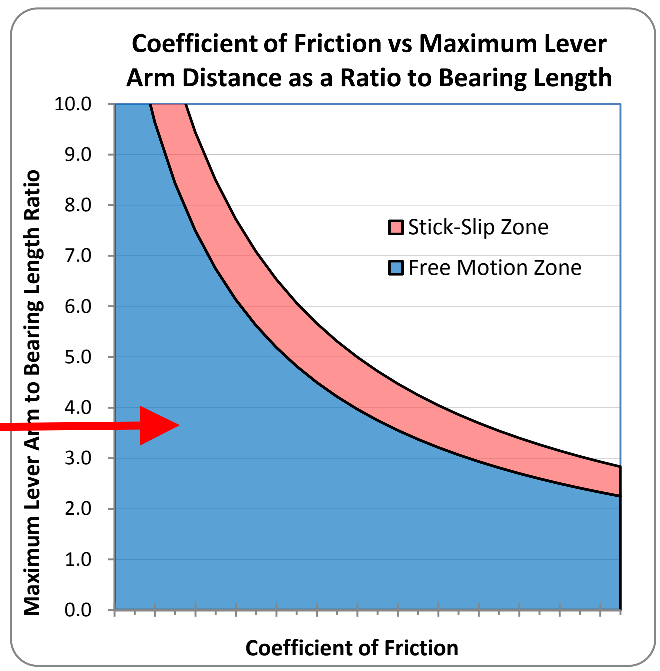

Figure 8: Coefficient of Friction vs. Max Lever Arm Distance for Typical Plane Bearing Systems

Figures 7 and 8, above, are not meant to be used as an engineering reference chart, rather, they should be used for illustration purposes only. The values for the X-Axis of both graphs have been removed to prevent accidental misuse. Instead, these images should be used to illustrate the point that all bearing systems have *two different* coefficients of friction (static & dynamic) and therefore, they also have two *different* Binding Ratios. Linear recirculating ball based systems often have static and dynamic coefficients of friction which are almost identical, so there is little change in the binding ratio. Plane bearings typically have a larger difference between the static and dynamic coefficient of friction, so there will be a larger variance in the binding ratios.

Stick-slip is often described as a temporary cycle of alternating rest and motion. Some systems experience stick-slip motion only at certain locations during the cycle. If this type of motion is repeatedly observed at the same location, then it is likely that an unknown force is acting upon the system at that location. The most common external forces are caused by misalignment of the linear rails, change of dimension for the rails, or an imperfection in the rail (which can also be caused by damage). These additional forces are not accounted for in Figure 5 and all of the equations shown in the white paper. These forces must be multiplied by the force of friction and then added to the resultant forces from the moment load. Looking back to Equation 10, above, it can be rewritten as Equation 16, below, to account for these additional forces. Provided that the applied force, F_1, is still larger than the frictional forces, then motion will still occur. However, if the sum of the frictional forces now exceeds the applied force, binding will occur.

Equation 16: $$Acceleration = 0 \rightarrow F_1 = \mu(F_2 + F_3) + \mu(other\ forces)$$ *Valid at Transition Point*

Since the system was likely in motion before these additional frictional forces were applied, the momentum of the system will help push through the momentary zone where these external forces are applied. As the system moves through this zone, the applied force will re-engage to cause a short burst of motion before resulting in binding again. The momentum will again help push the system through the binding where the cycle can repeat. This is stick-slip in its simplest form.

PRACTICAL COMPLICATIONS, LIMITATIONS & TROUBLESHOOTING

By far, the largest complication for working with the Binding Ratio, is that the actual Coefficient of Friction is hard to quantify and may change based upon environmental circumstances. To further complicate matters, some manufacturers either do not list the Coefficient of Friction in their product literature or may only list the dynamic Coefficient of Friction to make their products appear more favorable. Another issue with accurately using the Binding Ratio is the additional, unaccounted and often unpredictable forces caused by misalignment. In addition, some of the more advanced bearing materials, such as PBC's Frelon Gold, will actually have its Coefficient of Friction change based upon the load applied. A good rule of thumb is to take the expected coefficient of friction and to double it. This ensures there will be adequate safety factor within the design.

Another complication results from using the incorrect bearing length. Aside from mathematical and unit conversion errors, the most common problem is a result of using the wrong bearing length in the formulas shown earlier in this paper. This is very easy to do because the bearing length is <u>not</u> the overall bearing length, which is the most common assumption. Instead, bearing length means the length of the bearing carrying the load. In a ball bearing system, this is typically called the "load zone". Few manufacturers publish specifications as to the details of their load zone, so engineers have to guess a specific bearing ("load zone") length. Another common mistake happens for applications with multiple bearings on a single shaft. The correct bearing length is the center to center distance between the bearings and <u>not</u> the overall length (outside edge to outside edge length).

There are several steps an engineer can take to troubleshoot binding and stick-slip. At this point, we should note that this white paper is not intended to be an all-inclusive troubleshooting guide. Instead, it will list a few practical ideas which are based upon the theoretical principles previously discussed. There are five very simple concepts which, if implemented, should solve a binding/stick-slip problem for most applications. The five simple concepts are, in no particular order:

- Reduce Moment Arm Distance
- Increase Bearing Length
- Add a Counter Balance
- Remove External Forces
- Reduce Bearing Friction

The most logical change to prevent binding would be to reduce the moment arm distance. Reducing this distance will move the application out of the binding or stick-slip zone and bring it down into the smooth motion zone (reference: Figures 7 & 8, above). This is a great concept in theory; however, this is simply not an option for most applications as other system constraints prevent this distance from changing. The next suggested change would be to make the bearing length longer. This can be accomplished by switching to a longer bearing/carriage, increasing the spacing between multiple bearings or adding a second bearing to a single bearing system. This may be a solution for many applications; however, not all systems can allow for a longer bearing length. In this case, the next suggestion would be to try to add a counterbalance to reduce the moment which will reduce the resulting forces and consequently the frictional forces (reference: Figure 5, above). Again, this may not work for some applications as there may not be space to add a counterweight or overall system constraints prevent the additional weight or cost of the counterweight. A different solution would be to attempt to remove any external forces. These most often are a result of either misalignment or damage to a shaft/rail. Damaged shafts/rails may or may not be able to be fixed. They may have to be replaced in order to eliminate the additional forces encountered at the damaged portion. Attempting to fix a misaligned set of rails can often add considerable time and expense to the assembly process. This may be alright for a low quantity build, but is often unacceptable for mass production; which renders this option impractical for many applications. The final solution addressed by this paper would be to reduce the bearing friction. There are two primary ways to accomplish this: by adding or changing the type of lubrication to reduce the coefficient of friction or by changing the bearing type to a different style with a lower coefficient of friction.

Some applications will experience smooth motion in one direction and binding in the opposite direction. This is almost always a result of forces which were previously unaccounted for. Typically the forces are accounted for in one axis, but forces are seldom applied in only one axis. It only takes one axis having a force applied at a distance farther than the Binding Ratio distance to have the whole system experience stick-slip motion or complete binding. Troubleshooting this issue can be especially frustrating as the system seems to work half of the time. In this case, the most commonly applied solution is to increase the bearing length (either by increasing the distance between the bearings or switching to an extended length bearing.

The solutions provided in this white paper are listed in no particular order. Each application is unique and the solutions listed should be applied in whichever order is most beneficial to each specific application.

CONCLUSION

The Binding Ratio is a frequently misunderstood mechanical principle. Even though engineers are taught about 2:1 early on, misconceptions about the rule's implications still cause numerous stalling and application failures. After reading this white paper, you should have a more clear understanding of the mathematics behind the rule and a better comprehension of how to implement it. Just remember to be careful during the implementation of the Binding Ratio to ensure there is a proper safety factor, or else unpredictable and unaccounted forces may cause stick-slip and/or complete binding. This leads to system failure, and can create additional production costs, re-design time and lost profits. Keep your linear motion systems running by staying in the free motion zone and ahead of the Binding Ratio!

Further Information

If you're still having difficulties, contact a PBC Linear Application Engineer to discuss your application. You can contact an engineer directly by calling 1.800.962.8979 (from within the USA) or +1.815.389.5600 (from outside the USA). If you prefer e-mail, e-mail an engineer at: appeng@pbclinear.com

Version

This is version 2 of the published white paper. It has been updated 27 May 2010.

Update

As a result of questions from readers, Version 2 adds more information to the "Stick-Slip" and "Practical Complications, Limitations & Troubleshooting" sections. No other material changes were made.

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